

Warm-Up

Find all solutions. (Hint: Just Factor)

$$1. \ x^4 - 8x^2 + 7 = 0$$

$$\textcircled{1} \quad (x^2 - 7)(x^2 - 1) = 0$$

$$x = \pm\sqrt{7}, x = \pm 1$$

$$2. \ x^3 - 2x^2 + 5x - 10 = 0$$

$$\textcircled{2} \quad (x^2 + 5)(x - 2) = 0$$

$$x^2(x-2) + 5(x-2) = 0$$

$$x = \pm i\sqrt{5}, 2$$

$$3. \ x^3 + 125 = 0$$

$$a^3 + b^3 \Rightarrow (a+b)(a^2 - ab + b^2)$$

$$a=x \quad b=5 \quad (x+5)(x^2 - 5x + 25) = 0$$

$$\frac{5 \pm \sqrt{(-5)^2 - 4(1)(25)}}{2(1)}$$

$$\frac{5 \pm \sqrt{25 - 100}}{2}$$

$$\frac{5 \pm \sqrt{-75}}{2} = \frac{5 \pm \sqrt{-1}\sqrt{25}\sqrt{3}}{2}$$

$$\frac{5 \pm 5i\sqrt{3}}{2} - 5$$

Creating 'Least Degree' Equations from Zeros

- Determine the factor of each zero
- Multiply all of the factors together

Ex. 1 ZEROS: {-2, 1, 4}

$$\begin{aligned} & (x+2)(x-1)(x-4) \\ & x^2 - x + 2x - 2 \\ & (x^2 + x - 2)(x - 4) \\ & x^3 + x^2 - 2x - 4x^2 - 4x + 8 \\ & \text{---} \\ & \boxed{x^3 - 3x^2 - 6x + 8} \end{aligned}$$

Ex. 2 Zeros: {2, 3i, -3i}

$$(x-2)(x^2+9)$$

$$x^3 + 9x - 2x^2 - 18$$

$$\boxed{x^3 - 2x^2 + 9x - 18}$$

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = \pm\sqrt{-9}$$

$$x = \pm 3i$$

Ex. 3 Zeros $\{-4, 5, \sqrt{5}, -\sqrt{5}\}$

$$(x+4)(x-5)(x^2-5)$$

$x^2-5=0$
 $x^2=5$
 $x=\pm\sqrt{5}$

Double Root

- Means there is a turning point at that value
- Does not cross the x-axis

$$(x+3)(x+3)(x-1)$$

$$x = -3 \text{ (d.r.)}, x = 1$$

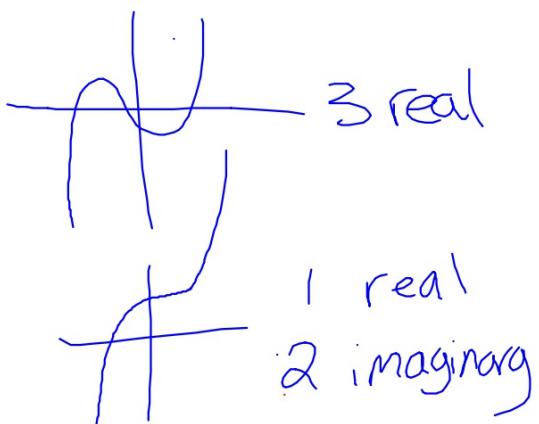
Fundamental Theorem of Algebra

Degree Tells us:

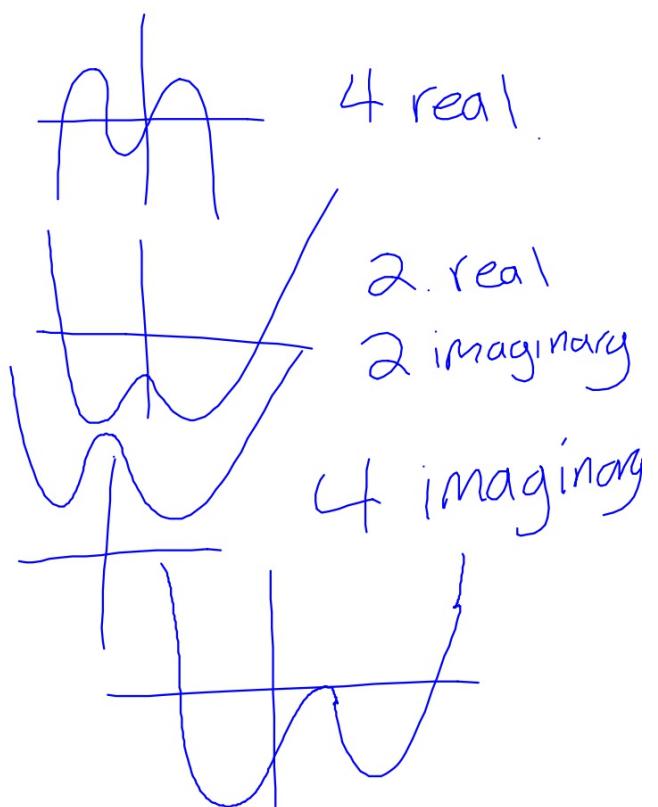
- - How many solutions
- - They could be real or imaginary

Possible Solutions

Cubic



Quartic



Ex. 4 $f(x) = -x^3 + x^2 - 2$

4

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -2 \\ \hline & 1 & -2 & 2 \\ \hline & -1 & 2 & -2 & 0 \end{array} \quad \checkmark$$

$$(x+1)(-x^2 + 2x - 2)$$

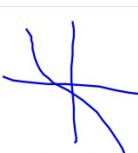
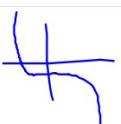
$$\frac{-2 \pm \sqrt{2^2 - 4(-1)(-2)}}{2(-1)}$$

$$\frac{-2 \pm \sqrt{4-8}}{-2} = \frac{-2 \pm \sqrt{-4}}{-2}$$

$$= \underline{\underline{-2 \pm 2i}}$$

$$x = 1 \pm i$$

End Behavior:



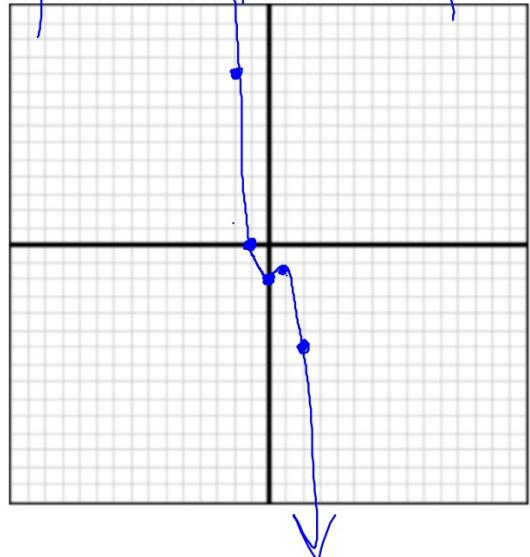
Factors: $(x+1)(-x^2+2x-2)$

Zeros: $-1, 1 \pm i$

Y- Intercept: $(0, -2)$

Relative Min: $(0, -2)$

Relative Max: $(\sqrt{6}, -1.85)$



Ex. 5 $f(x) = x^4 - 5x^3 + 7x^2 + 3x - 10$

-1, 2

(1) $\begin{array}{r} 1 \quad -5 \quad 7 \quad 3 \quad -10 \\ -1 \quad \quad 6 \quad -13 \quad 10 \\ \hline \end{array}$

(2) $\begin{array}{r} 1 \quad -6 \quad 13 \quad -10 \quad 0 \quad \checkmark \\ 2 \quad \quad -8 \quad 10 \\ \hline \end{array}$

$(x+1)(x-2)(x^2 - 4x + 5)$ Factors

$x = -1, 2, 2 \pm i$ Zeros

End Behavior:

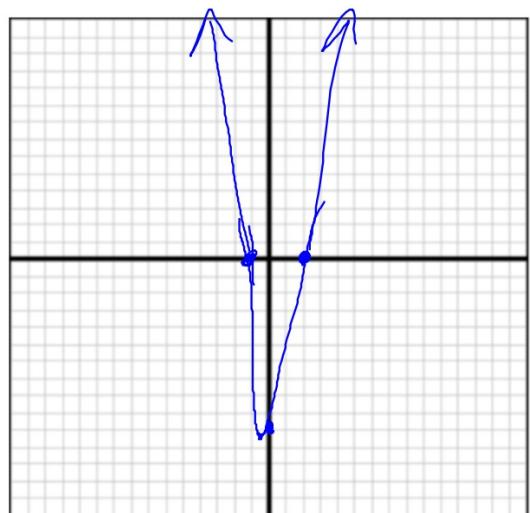
Factors:

Zeros:

Y- Intercept: $(0, -10)$

Relative Min: $(-1.6, -10.3)$

Relative Max: None



Friday's Test

- Everything from the Quiz
- Solving Polynomials (by factoring)
- Graphing Polynomials
- Creating Equations from Zeros

